4E4135

Roll No. ________

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4E4135

B. Tech. IV-Sem. (Main) Exam; April-May 2017

Electronics & Communication Engg.

4EC6A Mathematics - IV

Time : 3 Hours

Maximum Marks : 80

Min. Passing Marks : 24

Instructions to Candidates :-

*Attempt any **five questions**, selecting **one question** from **each unit**. All Questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.*
Units of quantities used / calculated must be stated clearly.
Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL   2. NIL

UNIT - I

1 (a) Define the operators : $\Delta, \nabla, \delta, \mu$. **2**

(b) Prove that :

$$u_0 + \frac{xu_1}{1!} + \frac{x^2u_2}{2!} + \frac{x^3u_3}{3!} + ... = e^x \left[u_0 + x\Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + ...\right]$$

8

(c) Prove that : $\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$. 6

OR

1 (a) Given the following data :

x:	10^o	20^o	30^o	40^o	50^o	60^o	70^o	80^o
y:	0.9848	0.9397	0.8660	0.7660	0.6428	0.5000	0.3420	0.1737

Evaluate :

(i) $y(25^o)$

(ii) $y(32^o)$

(iii) $y(73^o)$

8

(b) Apply Lagrange's formula to find $f(x)$ from the following data :

x:	0	1	4	5
$f(x)$:	4	3	24	39

Hence find f(3)

8

UNIT - II

2 (a) A slider in a machine moves along a fixed straight rod. Its distance $x(cm)$ along the rod is given below for various values of time $t(\text{secs.})$: Evalaute :

t:	0.0	0.1	0.2	0.3	0.4	0.5	0.6
x:	30.28	31.43	32.98	33.54	33.97	33.48	32.13

(i) $\frac{dx}{dt}$ for $t=0.1,\ t=0.3,\ t=0.5$

(ii) $\frac{d^2x}{dt^2}$ for $t=0.1,\ t=0.3,\ t=0.5$

8

(b) Use Milne's predictor-corrector method to obtain $y(0.4)$ and $y(0.5)$ for the following differential equation :

$\frac{dy}{dx}=2e^x-y$, given that

x:	0	0.1	0.2	0.3
y:	2	2.01	2.04	2.09

8

OR

2 (a) Evaluate $\int_{-1.6}^{-1} e^x \, dx$ by the

(i) Trapezoidal rule

(ii) Simpson's $\frac{1}{3}$ rule

(iii) Simpson's $\frac{3}{8}$ rule and compare your results with the exact value.

8

(b) If $\frac{dy}{dx} = x + y^2$, use Runge-Kutta method to find an approximate value of y for $x = 0.2$, given that $y = 1$ when $x = 0$ (take $h = 0.1$)

8

UNIT - III

3 (a) Establish the following differential formulae involving $J_n(x)$:

(i) $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}^{(x)}, \; n \geq 0;$

(ii) $\frac{d}{dx}\left[x^{-n} J_n(x)\right] = -x^{-n} J_{n+1}^{(x)}, \; n \geq 0.$

8

(b) Show that :

(i) $(2n+1)x P_n(x) = (n+1) P_{n+1}(x) + n P_{n-1}(x)$

(ii) $\int_{-1}^{1} x P_n(x) P_{n-1}(x)\, dx = \frac{2n}{(2n-1)(2n+1)}$. 8

OR

3 (a) State and prove orthogonal properties of Bessel's functions. 8

(b) Expand in a series of Legendre's polynomials :

$x^4 + 3x^3 - x^2 + 5x - 2$. 8

UNIT - IV

4 (a) There are three boxes containing respectively 1 white, 2 red and 3 black balls; 2 white, 3 red and 1 black ball; 3 white, 1 red and 2 black balls. A box is chosen at random and from it two balls are drawn at random. The two balls are one red and one white. Find the probability that these come from (i) the first box, (ii) the second box, (iii) the third box. 8

(b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution. Given that $P = \frac{1}{\sqrt{2\pi}} \int_0^Z e^{-\frac{1}{2}t^2} \cdot dt$, the values of Z corresponding to p=0.19 and p=0.42 are 0.50 and 1.40 respectively. 8

OR

4 (a) Razor blades are supplied by a manufacturing company in packets of 10. There is a probability of 1 in 100 blades to be defective. Using Poisson
2
distribution calculate the number of packets containing one defective blade, no defective blade and all defective blades in a consignment of 10,000 packets. **8**

(b) Two random variables have the least square regression lines with equations :

$3x+2y-26=0$ and $6x+y-31=0$. Find the mean values and the coefficient of correlation between x and y. **8**

UNIT - V

5 (a) Show that a necessary condition for $I=\int_{x_1}^{x_2} f(x,y,y')\,dx,\ y'=\frac{dy}{dx}$ to be an

3
extremum is that $\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0$. **8**

(b) Find the path on which a particle, in absence of friction, will slide from one fixed point to another point in the shortest time under the action of gravity. **8**

OR

5 (a) Find a function $y(x)$ for which

$$\int_0^1 \left[x^2 + (y')^2\right] dx \text{ is stationary given that } \int_0^1 y^2 dx = 2;\ y(0) = 0,\ y(1) = 0.$$

8

(b) Find the equations of the curves for which the functional

$$\int_0^1 \left[(y')^2 + 12xy\right] dx,\ y' = \frac{dy}{dx}$$

with $y(0) = 0$ and $y(1) = 1$ can be extremised.

8
