

3E1616

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B. Tech. III Semester (Main/Back) Examination-2014**Electronic Instrumentation & Control****3EI6 Advanced Engg. Mathematics-I****(Common to EC & EIC)****Time : 3 Hours****Maximum Marks : 80****Min. Passing Marks : 24****Instructions to Candidates:**

Attempt any **five** questions, selecting **one** question from **each** unit. All questions carry **equal** marks. (Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.)

Unit - I

1. a) Find the Laplace transform of
- $t^2 e^t \sin t$
 - $(t-1)^2 u(t-1)$ (8)
- b) Solve the differential equation using Laplace transform method.
- $$\frac{d^2 y}{dt^2} + m^2 y = a \cos nt$$
- given $y(0) = y'(0) = 0$. (8)

OR

1. ~~a)~~ Find the inverse Laplace transform of (8)
- ~~$\frac{s}{s^4 + 4a^4}$~~
 - ~~$\log\left(\frac{s+1}{s-1}\right)$~~
- ~~b)~~ Solve the partial differential equation using Laplace transform method.
- $$\frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2} \text{ Subject to}$$
- $u(0,t) = 0; u(2,t) = 0$. and $u(x,0) = 20 \sin 2\pi x, u_t(x,0) = 0$ (8)

Unit - II

2. a) Obtain the Fourier series for the function
 $f(x) = x^2, -\pi < x < \pi$ and deduce the result

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots \quad (8)$$

- b) Find the inverse Z-transform of

$$F(z) = \frac{1}{(z-5)^3}, |z| > 5 \quad (8)$$

OR

2. a) Find half - range sine series for $f(x) = x$ in $0 < x < 2$. (8)
 b) Obtain the constant term and the first three cosine terms in the Fourier series for y , where the values of y are given by the following table.

x	0	1	2	3	4	5	6
y	4	8	15	7	6	2	4

(8)

Unit - III

3. a) Obtain the Fourier transform of

$$f(x) = x^2, \text{ for } |x| \leq a$$

$$= 0 \text{ for } |x| > a \quad (8)$$

- b) Solve the boundary value problem for

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, \text{ using Fourier transform.}$$

$$\text{given } u(0, t) = u_0; t > 0;$$

$$u(x, 0) = 0 \text{ } x > 0 \text{ and } \frac{\partial u}{\partial x} \rightarrow 0 \text{ and as}$$

$$x \rightarrow \infty, u \rightarrow 0$$

(8)

OR

3. a) Find $F(x)$, if its Fourier sine transform is $\frac{1}{s} e^{-as}$. Also show that $\overline{F}\left(\frac{1}{s}\right) = \sqrt{\frac{\pi}{2}}$ (8)

- b) Find Fourier transform of $f(x) = \begin{cases} 1 & |x| \leq a \\ 0 & |x| > a \end{cases}$ and evaluate the integral

$$\int_{-\infty}^{\infty} \frac{\sin \lambda a \cos \lambda x}{\lambda} d\lambda$$

$$\text{Deduce the value of } \int_0^{\infty} \frac{\sin \theta}{\theta} d\theta$$

(8)

Unit - IV

4. a) Define the analytic function $f(z)$ and determine it if

$$f(z) = u + iv, \text{ where}$$

$$u = e^{2x} (x \cos 2y - y \sin 2y) \quad (8)$$

- b) Show that the transformation $W = \frac{2z+3}{z-4}$ maps the circle $x^2+y^2-4x=0$ into straight line $4u+3=0$ (8)

OR

4. a) Verify Cauchy's Theorem for $f(z)=z^3-iz^2-5z+2i$ if the contour C be the circle $|z-1|=2$ (8)

- b) Evaluate $\int_C \frac{\cos \pi z}{z^2-1} dz$ around the rectangles with vertices.

i) $2 \pm i, -2 \pm i$

ii) Vertices at $z = -i, z-i, z+i, i$. (8)

Unit - V

5. a) Obtain the Laurent's series expansion of

$$f(z) = \frac{e^z}{(z-1)^2} \text{ about } z=1. \quad (8)$$

- b) Find the residue of $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ at the poles in the finite part of z -plane. (8)

OR

5. a) Obtain the Taylor's series expansion of

$$f(z) = \frac{z^2-1}{(z+2)(z+3)} \text{ for the region } |z| < 2 \quad (8)$$

- b) Prove by contour integration

$$\int_0^\pi \frac{\log(1+x^2)}{1+x^2} dx = \pi \log_e 2 \quad (8)$$