3E1704

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B. Tech. (Sem. III) (Main) Examination, January - 2013

Petroleum Engg.

3PE1 Mathematics - III (Common with 3EC1, EIC, BM, AI, CR)

Time : 3 Hours] [Total Marks : 80

[Min. Passing Marks : 24

*Attempt any **five questions**, selecting **one question** from **each unit. All** questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly. Figures on the right hand side indicate full marks.*

Use of following supporting material is permitted during examination.

(Mentioned in form No. 205)

1. Non-programmable Scientific Calculator 2. Nil

UNIT - I

1 (a) The error function is defined as

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du$$

Find the Laplace transform of the error function and hence prove that

$$L\left[erfc\sqrt{t}\right] = \frac{1}{\sqrt{s+1}\left[\sqrt{s+1}+1\right]}$$

8

(b) Solve the ordinary linear differential equation

$$\frac{d^2y}{dt^2} + m^2 y = a\cos nt,\ t > 0$$

give that $y(0) = y_0$ and $y'(0) = y_1$. State the condition when the solution so obtained does not exist.

8

OR

1 (a) Find the inverse Laplace transform of the following :

(i) $\frac{1}{(8s-27)^{\frac{1}{3}}}$ (ii) $\frac{se^{-as}}{s^2-\omega^2}$

3+3

(b) Give the statement of convolution theorem on inverse Laplace transform. Justify it with a suitable example.

2

(c) The equation

$Y(x,s) = c_1 e^{xs} + c_2 e^{-xs} - \frac{x}{s^4}$ denotes the general solution of a PDE obtained by Laplace transform method. If the solution satisfies the initial conditions $y(x,0) = y_t(x,0) = 0$ and y remains finite for $x \to \infty, t \to \infty$ followed by $y(0,t) = 0$ then find the ultimate solution of the PDE.

2

UNIT-II

2 (a) Find the Fourier series for the function $f(x)$ defined by

$$f(x) = \begin{cases} \operatorname{sgn}(x), & x \neq 0 \text{ (lies between } -\pi \text{ and } \pi) \\ 0, & x = 0 \end{cases}$$

where $\operatorname{sgn}(x)$ denotes the signum function. 8

(b) Find the z-transform of the Dirac's delta function given by

$$\delta_n = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

4

(c) Find the inverse z-transform of $\log\left(\frac{z}{z+1}\right)$ using power series method. 4

OR

2 (a) Compute approximately, the Fourier coefficients a_0, a_1, a_2 and b_1, b_2 in the Fourier series expansion of function tabulated as follows :

x :	0	1	2	3	4	5
y :	9	18	24	28	26	20

Find the amplitude of the first harmonic.

10

(b) Find the inverse z-transform of $\frac{3z^2+2z}{z^2-3z+2}, 1<|z|<2$. **6**

UNIT-III

3 (a) Find the Fourier transform of the function $f(x)=|x|e^{-|x|}$. **6**

(b) Use the method of Fourier transform to obtain the displacement $u(x,t)$ of an infinite string, given that the string is initially at rest and the initial displacement is $f(x), -\infty<x<\infty$. Show that the solution can be represented by $u(x,y)=\frac{1}{2}\left[f(x+ct)+f(x-ct)\right]$. **10**

OR

3 (a) Use inverse cosine transform to find $f(x)$ in

$$\int_0^\infty f(x)\cos\lambda x\, dx = e^{-\lambda}$$

6

(b) Find the temperature distribution in a bar of length L, with its both ends and lateral surface insulated when the initial temperature in the bar is $f(x)$. Deduce the expression for temperature if $f(x)=x^2$ and $L=10$. **10**

UNIT-IV

4 (a) Determine where the function $f(z)=\begin{cases}\frac{z^2+3iz-2}{z+i} & , z\neq -i\\ 5 & , z=i\end{cases}$ is continuous ? Can the function be refined to make it continuous at $z=-i$? **8**

(b) Evaluate $\oint_C \frac{e^{3z}dz}{(z-\ln 2)^4}$ where C is the square with vertices at $\pm 1, \pm i$. **4**

(c) Find the analytic function $w = u + iv$, if the real part is given by $u = e^x \cos y$. 4

OR

4 (a) Prove that the cross-ratio of four points is invariant under a bilinear transformation. 6

(b) The ML-inequality of an analytic function $f(z)$ is

$$\left|\int_C f(z)\,dz\right| \le ML$$

where $|f(z)| \le M$ everywhere on C and L is the length of the curve. Use this inequality to find an upper bound for the absolute value of the integral $\int_c \left(e^z - \bar{z}\right) dz$ where c is the boundary of the triangle with vertices at $Z = 0, -4, 3i$. 10

UNIT - V

5 (a) Expand $f(z) = \sin z$ in a Taylor's series about $z = \frac{\pi}{4}$. 6

(b) Evaluate $I = \int_0^\pi \frac{a\,d\theta}{a^2 + \sin^2\theta}, a > 0$. 10

OR

5 (a) Show, by contour integration, that

$$\int_0^\infty \frac{\sin x}{x\left(a^2 + x^2\right)} dx = \frac{\pi}{2a^2}\left(1 - e^{-a}\right), a > 0.$$ 8

(b) Find the Laurent series of $f(z) = \frac{7z^2 + 9z - 18}{z^3 - 9z}$ in the regions :

(i) $0 < |z+3| < 3$

(ii) $3 < |z+3| < 6$

(iii) $|z+3| > 6$. 8
