3E1646

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B.Tech. (Sem.III) (Main/Back) Examination, 2015
Electrical Engineering
3EE6 Advanced Engineering Mathematics-I

Time : 3 Hours] [Total Marks : 80
[Min. Passing Marks : 26

Instructions to Candidates :
Attempt any five questions, selecting one question from each unit. All questions carry equal marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

UNIT-I

1. (a) Prove that $L\left\{\frac{\sin^2 t}{t}\right\} = \frac{1}{4}\log\left(\frac{s^2+4}{s^2}\right)$ and hence deduce the integral $\int_0^\infty \frac{\sin^2 t}{t^2}dt$ **(4+4=8)**

(b) Find the inverse Laplace transform of $\log\sqrt{1+\frac{9}{s^2}}$. **(8)**

OR

1. (a) Solve the following differential equation **(8)**

$$\left(D^2+9\right)y = \cos 2t \text{ with } y(0) = 1, y\left(\frac{\pi}{2}\right) = -1$$

(b) Solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = xt$ with boundary conditions $u(0) = 0, \left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$. **(8)**

UNIT-II

2. (a) Find the Fourier Transform of $f(x) = \begin{cases} 1-x^2, & |x|<1 \\ 0, & |x|>1 \end{cases}$ **(4+4=8)**

Also evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3}\cos\frac{x}{2}dx$

(b) Solve the following partial differential equation

$$\frac{\partial u}{\partial t} = \frac{c^2\partial^2 u}{\partial x^2}, t>0, -\infty<x<\infty$$

Satisfying u (x, 0) = f(x).

OR

2. (a) Find the Fourier sine and cosine transforms of **(4+4=8)**

$$f(x) = \begin{cases} x & 0<x\le 1 \\ 2-x & 1<x<2 \\ 0 & x\ge 2 \end{cases}$$

(b) Find the inverse DFT of the sequence **(8)**

$$\{G_j\} = \left\{0, \frac{3+i\sqrt{3}}{2}, \frac{3-i\sqrt{3}}{2}\right\}$$

UNIT - III

3. (a) Find the Fourier series for $f(x)= x+x^2, -\pi < x < \pi$ (8)

(b) The turning moment T units of the crank shaft of a steam engine is given for a series of the values of the crank angle θ in degrees: (8)

θ°	0	30	60	90	120	150	180
T	0	5224	8097	7850	5499	2626	0

Find the first four terms in a series of sine to represent T. Also find T when $\theta = 75°$.

OR

3. (a) Find a function y (x) for which $\int_0^1 \left[x^2+(y')^2\right]dx$ is stationary, given that $\int_0^1 y^2 dx = 2, y(0)=0=y(1)$. (8)

(b) Show that the arc length between two points in a plane is stationary when the curve joining them is a straight line. (8)

UNIT - IV

. (a) Show that the function $f(z)=e^{-z^{-4}}, z \neq 0, f(0)=0$ is not analytic at $z=0$, although C–R equations are satisfied at that point. (8)

(b) Find the bilinear transformation which maps the points z = 1, i, -1 respectively onto the points w = i, 0, -i. For this transformation, find the image of concentric circles $|z| = r, (r>1)$. (4 + 4 = 8)

OR

(a) Integrate $f(z) = x^2 + ixy$ from A (1, 1) to B (2, 4) along (4 + 4 = 8)

(i) The straight line joining the two points

(ii) The curve $c: x = t, y = t^2$

(b) Evaluate $\oint_c \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)}$ where c is a circle $|z| = 3$ by using Cauchy's integral formula. (8)

UNIT - V

(a) Develop the Taylor's series for the function $\frac{z^2-1}{(z+2)(z+3)}$ where $|z| < 2$ (8)

(b) Evaluate $\int_c \frac{\tan z}{z} dz$, where c is the contour $|z| = 2$. (8)

OR

(a) Evaluate $\int_c \frac{z^2 e^{zt}}{z^2+1} dz$ by residue theory, where c is the circle $|z| = 2$ and t is a quantity independent of z. (8)

(b) Evaluate $I = \int_0^\pi \frac{a\,d\theta}{a^2+\sin^2\theta}, \; a > 0.$ (8)