

<b>4E4161</b>	Roll No.	<b>4E4161</b>	Total No of Pages: <span style="border: 1px solid black; padding: 2px;">4</span>
<b>B. Tech. IV Sem. (Main/Back) Exam., June/July-2014</b> <b>Computer Science and Engineering</b> <b>4CS2A Discrete Mathematical Structures</b> <b>Common with IT</b>			

**Time: 3 Hours****Maximum Marks: 80**  
**Min. Passing Marks: 24****Instructions to Candidates:-**

*Attempt any five questions, selecting one question from each unit. All Questions carry equal marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.*

*Units of quantities used/ calculated must be stated clearly.*

*Use of following supporting material is permitted during examination.*

*(Mentioned in form No.205)*

1. \_\_\_\_\_ NIL \_\_\_\_\_

2. \_\_\_\_\_ NIL \_\_\_\_\_

**UNIT – I**

Q.1 (a) (i) Prove, for finite sets A and B;

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) \quad [4]$$

(ii) In a class of 50 students, 15 play Tennis, 20 play Cricket and 20 play Hockey, 3 play Tennis and Cricket, 6 play Cricket and Hockey, and 5 play Tennis and Hockey. 7 play no game at all. How many play Cricket, Tennis and Hockey? [4]

- (b) (i) Define floor function and ceiling function with example. [4]  
(ii) "If  $f$  and  $g$  are two bijections such that  $g \circ f$  is defined then  $g \circ f$  is also a bijection." Prove it. [4]

**OR**

- Q.1 (a) (i) If  $f: A \rightarrow B$  be one-one onto then the inverse map of  $f$  is unique. Prove it. [4]  
(ii) Show that set of even positive integers is a countable set. [4]  
(b) State and prove the Pigeonhole and Generalized Pigeonhole Principles. [8]

## **UNIT – II**

- Q.2 (a) Define the following with example:-  
(i) Equivalence relation [2]  
(ii) Congruence relation [2]  
(iii) Partial order relation [2]  
(iv) Total order relation [2]  
(b) Explain closure of relations. Let  $A = \{1,2,3,4\}$  and let  $R = \{(1,2) (2,3) (3,4), (2,1)\}$  be a relation on  $A$ . Find the transitive closure of  $R$  using Warshall's algorithm. [8]

**OR**

- Q.2 (a) Let  $A = \mathbb{Z}$ , the set of integers relation  $R$  define in  $A$  by  $aRb$  as "a is congruent to  $b \pmod{2}$ ". Prove that  $R$  is an equivalence relation. [8]  
(b) (i) Show that  $(\mathbb{Z}^+, \text{divisibility})$  is a poset. [4]  
(ii) Compute the number of partitions of a set with four elements. [4]

**UNIT – III**

- Q.3 (a) (i) Prove  $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ . [4]
- (ii) Write contrapositive, converse and inverse of the statement.  
 “The home team wins whenever it is raining”. Also construct the truth table for each statement. [4]
- (b) (i) Prove that the linear search algorithm works correctly for every  $n \geq 0$ . [4]
- (ii) Sort the list  $X = \{64, 25, 12, 22, 11\}$  using selection sort algorithm. [4]

**OR**

- Q.3 (a) Every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps. Prove this by using principle of complete induction. [8]
- (b) Prove the implication “If  $n$  is an integer not divisible by 3, then  $n^2 \equiv 1 \pmod{3}$ ”. [8]

**UNIT – IV**

- Q.4 (a) Define the following with example:-
- (i) Complete graph, [2]
- (ii) Bipartite graph, [2]
- (iii) Isomorphic graph, [2]
- (iv) Planar graph, [2]
- (b) (i) Suppose that  $G = (V, E)$  be a graph with  $K$  – component, where each component is a tree. Derive a formula in terms of  $|V|$ ,  $|E|$  and  $K$ . [4]
- (ii) Let there is a tree with  $n$ -vertices of degree 1, 2 vertices of degree 2, 4 vertices of degree 3 and 3 vertices of degree 4. Obtain the value of  $n$ . [4]

**OR**

- Q.4 (a) Prove that a simple graph with  $n$  vertices and  $k$  components can have almost  $\frac{1}{2}[(n-k)(n-k+1)]$  edges. [8]
- (b) Show that the complete bipartite graph  $K_{3,3}$  is a non – planar graph. [8]

**UNIT – V**

- Q.5 (a) (i) Explain Tautology, contradiction and contingency. [2]  
(ii) Show that  $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$  is a tautology. [2]  
(iii) Show that  $p \wedge \sim p$  is a contradiction. [2]  
(iv) Show that  $(p \rightarrow q) \wedge (p \vee q)$  is a contingency. [2]
- (b) Test the validity of the following argument:  
If I like mathematics, then I will study.  
Either I study or I fail.  
Therefore, if I fail then I do not like mathematics. [8]

**OR**

- Q.5 (a) Check the validity of the following argument:  
Lions are dangerous animals.  
There are lions.  
Therefore, there are dangerous animals. [8]
- (b) (i) ~~Define the quantifiers. Explain types of quantifiers.~~ [4]  
(ii) Over the universe of animals, let  
 $P(x)$  : x is a whale ;  $Q(x)$  : x is a fish  
 $R(x)$  : x lives in water.  
Translate the following into English  
 $\exists x (\sim R(x))$   
 $\exists x (Q(x) \wedge \sim P(x))$   
 $\forall x (P(x) \wedge R(x)) \rightarrow Q(x)$  [4]